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A CLOSE MATCH: THE SERIES

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ABSTRACT: In this scenario, the final word of the series is employed to draw near to an alternate series. The phrase "correction term" is becoming increasingly common. The correction term is an essential component of series estimation**.**

Keywords: Correction function, error function, remainder term, alternating series, rational approximation, Dirichlet's series.

1. INTRODUCTION

In the 1400s, a great scientist named Madhava proposed the adjustment function for the pi series. It's from the Madhava series.

$$C = \frac{4d}{1} - \frac{4d}{3} + \frac{4d}{5} - \cdots\cdots + (-1)^{n-1}\frac{4d}{2n-1} + (-1)^{n}\frac{4d(2n)/2}{(2n)^2+1}$$

how big the diameter of a circle with a diameter of d is.

The word that sticks out in particular is

$(-1)^n$ 4d G_n

Where $G_n = \frac{(2n)/2}{(2n)^2+1}$ is the word that should be modified. By including the adjustment term, the value of C can be computed more precisely.

RATIONAL APPROXIMATION OF ALTERNATING SERIES $\sum^{\infty}$ $(-1)^{n-1}$

$n=1$ an^2+bn+c

where a,b,c $\in$ R with $a \neq 0$ and $\sqrt{b^2-4ac} \neq 2a$.

The alternating series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{an^2+bn+c}$ satisfies the conditions of alternating series test and so it is convergent.

Theorem

The correction function for the alternating series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{an^2+bn+c}$ where a,b,c $\in$ R with $a \neq 0$ is $G_n = \frac{1}{[2an^2+(2b+2a)n+(2c+b+2a)]}$

Proof

If G_n is the correction function after n terms of the series ,then

we have $G_n + G_{n+1} = \frac{1}{an^2+(2a+b)n+a+b+c}$

The error function is $E_n = G_n + G_{n+1} - \frac{1}{an^2+(2a+b)n+a+b+c}$

Let $G_n(r_1, r_2) = \frac{1}{[2an^2+(4a+2b)n+(2a+2b+2c)]-(r_1 n+r_2)}$ where $r_1, r_2 \in$ R and n is fixed.

Then error function $|E_n(r_1, r_2)|$ is minimum for $r_1 = 2a$, $r_2 = b$

Hence for $r_1 = 2a$, $r_2 = b$, both G_n and E_n are functions of a single variable n.

Thus the correction function for the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{an^2+bn+c}$ is

$$G_n = \frac{1}{[2an^2+(2b+2a)n+(2c+b+2a)]}$$

The corresponding error function is

$$|E_n| = \frac{|(b^2-4ac)-4a^2|}{[2an^2+(2b+2a)n+(2c+b+2a)][(2an^2+(2b+6a)n+(6a+3b+2c)][(an^2+(2a+b)n+(a+b+c)]}$$

Hence the proof.

REMARK

Gn does not, without a doubt, exactly match the value of the term (n+1).

2.APPLICATION

The series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2} = \eta(2)$

Using a calculator, we get the answer (2) = 0.8224670334.

Getting the series corrected in some way is essential.

$$G_n = \frac{1}{2n^2+2n+2}$$

After the correction function was applied, the series approximation for n = 10 is displayed in the next section.

Number of terms	S_n	$S_n + (-1)^n G_n$
10	**0.8**179621756	**0.822466**66801

THE ALTERNATING SERIES

The alternating series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n(n+1)}$ is convergent and converges to 2log2-1.

We have 2log2-1 = 0.3862943611, using a calculator.

The correction function for the series is $G_n = \frac{1}{2(n+1)^2+1^2}$

After the correction function was applied, the series approximation for n = 10 is displayed in the next section**.**

Number of terms	S_n	$S_n + (-1)^n G_n$
10	0.**38**21789321	**0.386**3283098

3. CONCLUSION

When an adjustment function is used, both the estimate and the sum of the series improve.

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