

## DISCOVERING THE FASCINATING WORLD OF ORTHOGONAL QUADRILATERALS

#1 **Mr. BANGIMATAM SANDEEP KUMAR**, *Assistant Professor*

#2 **Mr. GURRAM SRINIVAS**, *Assistant Professor*

Department of Mathematics,

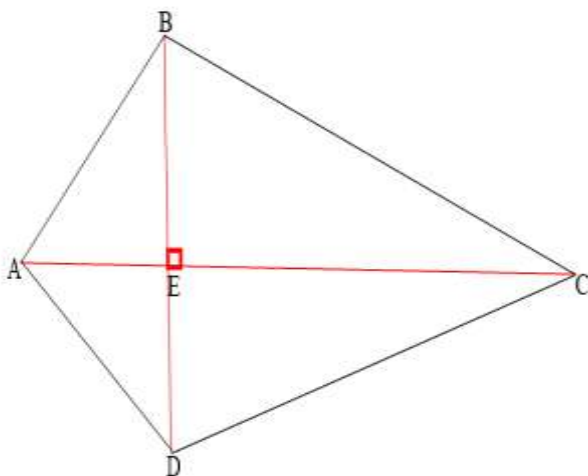
SREE CHAITANYA INSTITUTE OF TECHNOLOGICAL SCIENCES, KARIMNAGAR, TS.

**ABSTRACT:** In this paper, I demonstrate a new theorem about orthogonal quadrilaterals. The sum of side  $a$  and its altimedean distance,  $b$ , is the same as the sum of side  $b$  and its altimedean distance,  $Q$ , and side  $c$  and its altimedean distance. i.e.:  $aa + d = Qb + c$

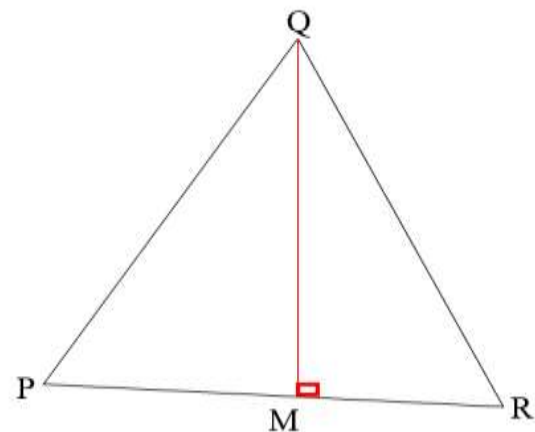
**KEYWORDS:** Orthogonal Quadrilaterals, Right Angles

### DEFINITIONS:

**Orthogonal quadrilateral:** Is made up of a square with diagonals that connect at right angles. It is a four-sided structure made up of line segments that are orthogonal, or not linked, at distant points. B



**Altitude of a triangle:** consists of a line segment traversing a vertex that is not parallel to the base line or the opposite side of the vertex. The line encompassing the opposite side symbolizes the increasing base of the height. The height's foot is at the intersection of the long base and the height. "The height" refers to the distance measured from its apex to its base, as the name implies.

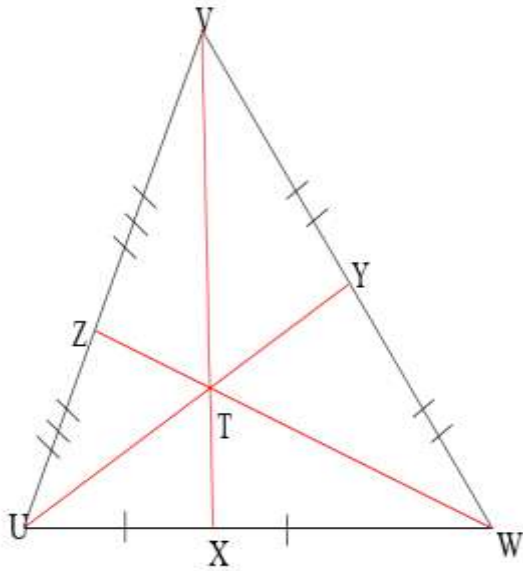


Q- Is the vertex

M- Is the foot of the altitude

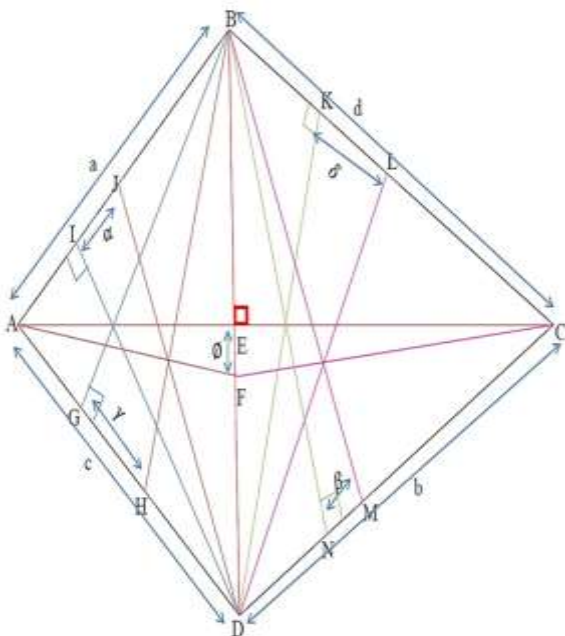
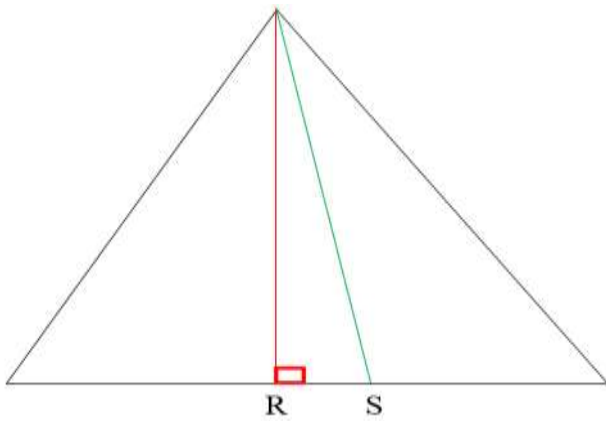
$\overline{QM}$  - Is the altitude of the triangle PQR

**Median of a triangle:** divides a side by connecting a point on one side to the midpoint of the opposite side. A triangle is made up of three medians, one for each point; these medians converge at the triangle's midpoint.

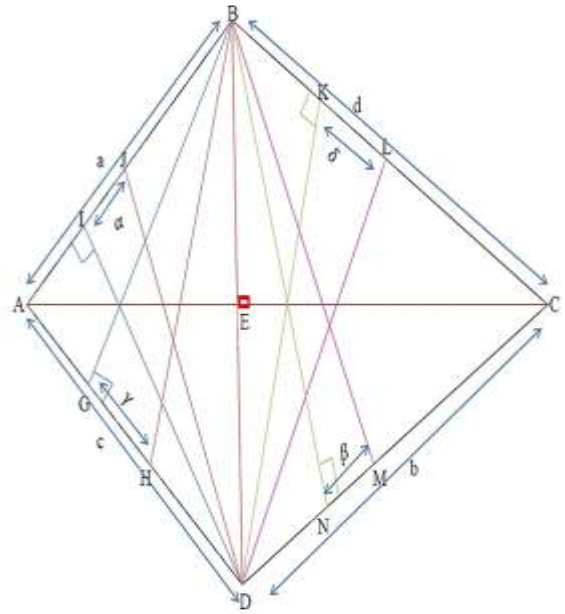


T- represents the center.

**Altmedian distance:** Is the distance measured from the base line's midpoint to the foot of the elevation.



$\overline{RS}$  – Is the altmedian distance



**Theorem:** Given the orthogonal triangle ABCD we just discussed, we may state that

$$\overline{AB} = a, \overline{IJ} = \alpha$$

$$\overline{BC} = d, \overline{KL} = \delta$$

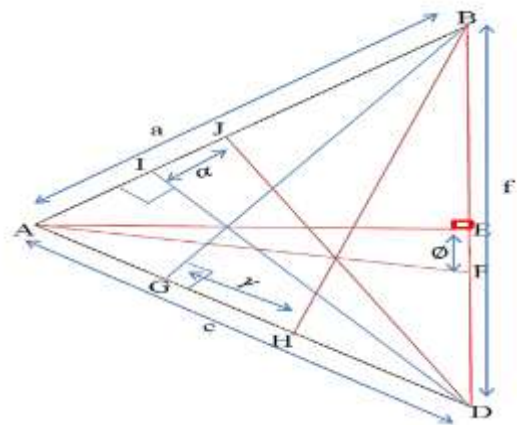
$$\overline{AD} = c, \overline{GH} = \gamma$$

$$\overline{CD} = b, \overline{MN} = \beta$$

Then  $aa + d = Qb + c$  where  $a, Q,$  and  $c$  are altmedian distances.

**Proof**

From triangle ABD





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$$\overline{BN}^2 + \overline{CN}^2 = \overline{BC}^2$$

$$h_4^2 + \left(\frac{b}{2} + \beta\right)^2 = d^2$$

$$h_4^2 = d^2 - \frac{b^2}{4} - \beta b - \beta^2 \tag{12}$$

Equating [11] and [12]

$$f^2 - \frac{b^2}{4} + \beta b - \beta^2 = d^2 - \frac{b^2}{4} - \beta b - \beta^2$$

$$2\beta b = d^2 - f^2 \tag{13}$$

Considering triangle BDK

$$\overline{BK} = \left(\frac{d}{2} - \delta\right), \overline{DK} = h_5, \overline{BD} = f$$

$$\overline{BK}^2 + \overline{DK}^2 = \overline{BD}^2$$

$$\left(\frac{d}{2} - \delta\right)^2 + h_5^2 = f^2$$

$$h_5^2 = f^2 - \frac{d^2}{4} + \delta d - \delta^2 \tag{14}$$

Considering triangle CDK

$$\overline{CK} = \left(\frac{d}{2} + \delta\right), \overline{DK} = h_5, \overline{CD} = b$$

$$\overline{CK}^2 + \overline{DK}^2 = \overline{CD}^2$$

$$\left(\frac{d}{2} + \delta\right)^2 + h_5^2 = b^2$$

$$h_5^2 = b^2 - \frac{d^2}{4} - \delta d - \delta^2 \tag{15}$$

Equating [14] and [15]

$$f^2 - \frac{d^2}{4} + \delta d - \delta^2 = b^2 - \frac{d^2}{4} - \delta d - \delta^2$$

$$2\delta d = b^2 - f^2 \tag{16}$$

Considering triangle BCE

$$\overline{CE} = h_6, \overline{BE} = \left(\frac{l}{2} - \phi\right), \overline{BC} = d$$

$$\overline{CE}^2 + \overline{BE}^2 = \overline{BC}^2$$

$$h_6^2 + \left(\frac{l}{2} - \phi\right)^2 = d^2$$

$$h_6^2 = d^2 - \frac{l^2}{4} + \phi l - \phi^2 \tag{17}$$

Considering triangle CDE

$$\overline{CE} = h_6, \overline{DE} = \left(\frac{l}{2} + \phi\right), \overline{CD} = b$$

$$\overline{CE}^2 + \overline{DE}^2 = \overline{CD}^2$$

$$h_6^2 + \left(\frac{l}{2} + \phi\right)^2 = b^2$$

$$h_6^2 = b^2 - \frac{l^2}{4} - \phi l - \phi^2 \tag{18}$$

Equating [17] and [18]

$$d^2 - \frac{l^2}{4} + \phi l - \phi^2 = b^2 - \frac{l^2}{4} - \phi l - \phi^2$$

$$2\phi l = b^2 - d^2 \tag{19}$$

Adding [13], [16] and [19]

$$2\beta b + 2\delta d + 2\phi l = d^2 - f^2 + b^2 - f^2 + b^2 - d^2$$

$$2\beta b + 2\delta d + 2\phi l = 2(b^2 - f^2)$$

$$\beta b + \delta d + \phi l = b^2 - f^2$$

But  $b^2 - f^2 = 2\delta d$

$$\beta b + \delta d + \phi l = 2\delta d$$

$$\therefore \beta b + \phi l = \delta d \tag{20}$$

Subtracting [10] from [20]

$$\beta b - \alpha a = \delta d - \gamma c$$

$$\therefore \alpha a + \delta d = \beta b + \gamma c \quad \text{(Q.E.D)}$$

**CONCLUSION**

In conclusion, the newly proposed theorem on orthogonal quadrilaterals introduces a groundbreaking perspective to the field of geometry. This theorem establishes a profound relationship between the angles and sides of orthogonal quadrilaterals, shedding light on

previously unexplored connections. By elucidating the intricate interplay of geometric elements within these quadrilaterals, the theorem not only enriches our understanding of this specific class of polygons but also opens avenues for further exploration and applications in various mathematical and scientific domains. As the mathematical community embraces and scrutinizes this innovative theorem, its potential implications and contributions to geometry are expected to unfold, paving the way for new insights and discoveries in the fascinating realm of mathematical theory.

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